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HEAT AND MASS TRANSFER UNDER COOLING OF FLOWING WATER SHEETS BY AN AIR COUNTERFLOW

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On the basis of the system of differential equations in partial derivatives, the boundary-value problem of heat and mass transfer under cooling of flowing water sheets by a turbulent air counterflow has been formulated. Its approximate analytical solution is given. From the point of view of the accepted model of interaction of flows, it has been shown that the obtained theoretical values of the heat and mass transfer coefficients are in good agreement with the experiment.

The heat and mass transfer between liquid and gas flows is widely used in technology and is of great importance for solving many energy and ecological problems: cooling and heating of gases and liquids in various heaters and contact heat exchangers, air conditioning, circulating water supply of enterprises, etc. The case of water cooling after the heat-exchangers at thermal and nuclear power stations in cooling towers using a large quantity of water and having considerable sizes occupies a special place.

Despite the fact that cooling towers have been used for a long time, the mechanism of the processes proceeding in them is still not clearly understood. The state of the calculation of these processes is given in [1]. Analysis of the calculating techniques shows that they are based on considerable simplifications. At best, the authors go from three equations in partial derivatives describing the joint processes of heat and mass transfer to the three ordinary differential equations first used in the 1930s by Proskuryakov. But in the majority of cases even simpler approaches (Merkel's method and its modification and, in particular, Berman's method [2]) are used. The employment of these methods leads to the necessity of using empirical relations for the heat and mass coefficients or, in the most simplified form — to volume coefficients. Usually, one volume coefficient is used and the second one is found from the analogy between the processes of heat and mass transfer by the Lewis equation. Such approaches do not provide complete information about the process. For instance, on the basis of the numerical solution of ordinary differential equations, in [3] the heat and mass transfer under heating of laminar flowing water films by a turbulent hot gas flow in contact heat exchangers and in [4] evaporative cooling of flowing-down films at a laminar flow of phases in cooling towers were considered.

The most characteristic and effective regimes of water–air interaction in cooling towers is the counterflow regime in general and the film regime for a liquid. And the liquid flow thereby occurs under laminar-wave conditions, which provides the maximum residence time of the liquid in the contact zone of its cooling.

The elementary cell of the liquid–gas interaction represents a vertical (or slightly deviated from vertical) flat surface (packing) by which the water flows in a thin sheet and the air goes up, contacting at the interface the liquid and cooling it. In so doing, the air is heated and humidified due to the water evaporation. Of greatest practical importance is the turbulent air flow. The maximum air-flow rates in cooling towers are 4–5 m/sec, which corresponds to the conditions of weak hydrodynamic action [5]. Under these conditions, the hydrodynamic parameters of the film are practically independent of the turbulent gas flow and remain the same as at a free flow of films. On the contrary, the gas hydrodynamics through the velocity profile strongly depends on the characteristics of the liquid film. The above conditions are observed at an average velocity of the gas of up to 7–8 m/sec; at a larger velocity the flooding regime arises and then an upward counterflow of the liquid film appears under the action of the shear stress on the side of the gas. In so doing, the hydrodynamic and thermal parameters of the film strongly depend on the gas parameters.

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Let us formulate the boundary-value problem for the heat and mass transfer under the following assumptions: we neglect the diffusion heat conduction, the thermal diffusion, and the transverse flow of the mass because of their smallness; the physical properties of the water and the air are constant because of their small temperature change; the film surface is flat.

The coordinate system was chosen so that its origin is at the film–gas interface, the x-axes for the film and the gas are directed towards each other and are related by the relation $x_g = l - x_1$, and the y-axes are directed, respectively, to the liquid film and the gas phase. Then the heat and mass transfer in the system "laminar water film–turbulent air counterflow" in the approximation of the boundary layer can be described by differential equations in partial derivatives in the conjugate formulation:

for the heat transfer in the flowing film

$$u \frac{\partial t}{\partial x} = a \frac{\partial^2 t}{\partial y^2}, \tag{1}$$

for the heat transfer in the turbulent gas flow

$$\rho_g c_{pg} u_{g0} \frac{\partial T}{\partial x_g} = \frac{\partial}{\partial y_g} \left((\lambda_g + \lambda_{t,g}) \frac{\partial T}{\partial y_g} \right), \tag{2}$$

and the mass transfer in the gas

$$u_{g0} \frac{\partial C}{\partial x_g} = \frac{\partial}{\partial y_g} \left((D + D_t) \frac{\partial C}{\partial y_g} \right), \tag{3}$$

which should satisfy the following initial and boundary conditions:

$$\begin{aligned} & x = 0, \quad t = t_{in}, \\ & x = l, \quad T = T_{in}, \quad C = C_{in}; \\ & y = \delta, \quad \frac{\partial t}{\partial y} = 0; \\ & y = h - \delta, \quad \frac{\partial T}{\partial y_g} = \frac{\partial C}{\partial y_g} = 0; \\ & y = 0, \quad t = T = t_i, \quad C = C_i, \quad q_{liq} = q_g + r m_v. \end{aligned} \tag{4}$$

According to the last condition in (4), the thermal flow from the film at the interface is formed by the flow due to the convection to the gas and the heat of evaporation of the liquid.

At the interface, an equilibrium vapor concentration C_i at a given pressure is attained:

$$C_i = f(t_i). \tag{5}$$

In the general case, the function f in (5) is nonlinear. Thus, the system of equations (1)–(3) with conditions (4) and (5) is interrelated and nonlinear.

The solution of Eq. (1) for a laminar flow with constant boundary conditions and an arbitrary velocity profile u in the film (e.g., a parabolic one) is given in [6]. For a turbulent liquid flow the solution is complicated. Note also that the problem should be solved in two phases where their output characteristics are not known in advance and are to be determined in the process of solution. Strictly speaking, one should add to the system of equations (1)–(3) the equations describing the hydrodynamics of the air counterflow interacting with the liquid film. However, this is impossible because of the lack of adequate data on the conditions of the interaction of the flows at the interface and a rigorous theory of turbulence (the more so for a two-phase flow). Therefore, for the velocity profile in Eqs. (2) and (3), the semiempirical data taking into account the interaction of the liquid and gas flows will be used.

The aim of the present investigation is to obtain more detailed information about the process of heat and mass transfer, compare the theoretical and experimental values of the heat and mass-transfer coefficients, and elucidate the influence of the waves on the film surface.

On going to dimensionless coordinates by the relations

$$u = u_i\,(1-\eta^2)\,,\quad \eta = \frac{y}{\delta}\,,\quad \xi = \frac{x}{\delta}\,\frac{1}{\mathrm{Pe_f}}\,,\quad \mathrm{Pe_f} = 1.5\mathrm{Re_f}\mathrm{Pr}\,,\quad \mathrm{Re_f} = \frac{u_{\mathrm{av}}\delta}{\nu}\,,$$

$$u_i = 1.5u_{\mathrm{av}}\,,\quad u_{\mathrm{av}} = 3^{-1/3}\,(\nu g)^{1/3}\,\mathrm{Re_f^{2/3}} = 0.693\,(\nu g)^{1/3}\,\mathrm{Re_f^{2/3}}\,,\quad \mathrm{Re_f} < 400$$

Eq. (1) transforms to the form

$$(1-\eta^2)\,\frac{\partial t}{\partial \xi} = \frac{\partial^2 t}{\partial \eta^2}\,, \tag{6}$$

where a parabolic velocity profile in the liquid film is used.

To solve Eqs. (2) and (3), one should know the laws of change in the velocity u_{g0} and the turbulent heat conductivity $\lambda_{\mathrm{t,g}}$ and diffusion D_{t} coefficients. In general, the gas flow moves relative to the film surface at a mean velocity $\bar{u}_{\mathrm{g0}} = \bar{u}_{\mathrm{g}} + u_i$. The hydrodynamic parameters of the gas strongly depend on the characteristics of the liquid film, which affects the velocity profiles in the gas and the friction drag [7, 8]. With increasing spray density the degree of filling of the velocity profiles decreases compared to the flow in smooth pipes; they "elongate" and become similar to the profiles of the laminar flow or the non-Newtonian liquid flow.

According to [7], the velocity profiles of the turbulent gas flow can be described fairly exactly by the power law

$$\frac{u_{\mathrm{g0}}}{u_{\mathrm{g0max}}} = \left(\frac{1.1y}{h-\delta}\right)^{s}\,, \tag{7}$$

where

$$s = 0.85\,\sqrt{\lambda_0}\,. \tag{8}$$

The exponent s increases with increasing spray density and decreases with increasing $\mathrm{Re_{g0}}$ and $s > 1/7$, and the drag coefficient for the channel being sprayed in (8) is determined according to [8] by the Darcy–Weisbach formula

$$\Delta p = \lambda_0\,\frac{l\rho_{\mathrm{g}}\bar{u}_{\mathrm{g0}}^2}{8\,(h-\delta)}\,.$$

For the laminar-wave liquid flow ($7.5 \le \mathrm{Re_f} \le 400$), the dependence

$$\frac{\lambda - \lambda_0'}{\lambda_0'} = 4.385\cdot10^{-3}\,\mathrm{Re_f^{2/3}} \tag{9}$$

has been found. The drag coefficient for the smooth pipe λ_0' is determined by the Blasius formula.

The value of the maximum velocity on the channel axis is obtained from the relation

$$\frac{u_{\mathrm{g0max}} - \bar{u}_{\mathrm{g0}}}{\bar{u}_{\mathrm{g0}}} = 1.44\,\sqrt{\lambda_0}\,, \tag{10}$$

and the wavelength-mean (or effective) velocity of the liquid film surface layer u_i is found by the data of [9].

Formulas (7)–(10) completely describe the velocity profile of the gas phase. It should be noted that in the known works the change in this velocity profile under interaction with the running-down liquid film is ignored.

If we introduce into (2) and (3) the dimensionless quantities

$$\eta_g = \frac{y}{h - \delta}, \quad \xi_g = \frac{x_g}{\mathrm{Pe}_g\,(h - \delta)},$$

$$\mathrm{Pe}_g = \frac{1.1^s\, u_{g0\max}\,(h - \delta)}{a_g} = \mathrm{Re}_{g0\max}\mathrm{Pr}_g\,, \quad \mathrm{Re}_{g0\max} = \frac{1.1^s\, u_{g0\max}\,(h - \delta)}{\nu_g}\,,$$

then on the usual going to the turbulent viscosity they will be rewritten as follows:

$$\eta_g^s \frac{\partial T}{\partial \xi_g} = \frac{\partial}{\partial \eta_g}\left[\left(1 + \frac{\nu_t}{\nu}\frac{\mathrm{Pr}}{\mathrm{Pr}_t}\right)\frac{\partial T}{\partial \eta_g}\right], \tag{11}$$

$$\eta_g^s \frac{\partial C}{\partial \xi_g} = \frac{1}{\mathrm{Le}}\frac{\partial}{\partial \eta_g}\left[\left(1 + \frac{\nu_t}{\nu}\frac{\mathrm{Pr}_d}{\mathrm{Pr}_{d,t}}\right)\frac{\partial C}{\partial \eta_g}\right]. \tag{12}$$

To write the expression for the turbulent viscosity, we make use of the Millionshchikov model, which is of interest because of its relative simplicity, consistency, and the possibility of obtaining analytical relations for the heat-exchange characteristics in explicit form. As was shown in [6], the application of this model and its generalizations in various cases of heat exchange (without the phase transition, with evaporation, under the action of the shear stress on the side of the gas) provides a good agreement between calculated and experimental data. A detailed analysis of the more complex models is also given in [6].

Let us write the dependences for ν_t/ν in the form

$$\begin{aligned} &\frac{\nu_t}{\nu} = 0\,, && \eta_g < \frac{7.8}{h^+ - \delta^+}\,; \\ &\frac{\nu_t}{\nu} = 0.39\,(1 - \eta_g)\,[\eta_g\,(h^+ - \delta^+) - 7.8]\,, && \frac{7.8}{h^+ - \delta^+} \le \eta_g \le 1\,, \end{aligned} \tag{13}$$

where $h^+ = hV_{g0}/\nu_g$, $\delta^+ = \delta V_{g0}/\nu_g$ are, respectively, the dimensionless half-width and thickness of the film; V_{g0} is the dynamic gas velocity ($V_{g0} = u_{g0}\sqrt{\lambda_0/8}$); and the coefficient 7.8 is the dimensionless thickness of the viscous sublayer.

Relation (13) takes into account the turbulence damping as the free surface is approached, which agrees with the current notions. In dependences (11) and (12), in view of (13) the mean value of $\mathrm{Pr}_t = \mathrm{Pr}_{d,t} = 0.9$ throughout the gas layer thickness is assumed.

The values of the Pr and Pr_d numbers in (11) and (12) for the gas phase are of interest. For instance, while for dry air the value of Pr = 0.722 is constant, for humid air on the saturation line at temperatures of 0–50°C it smoothly increases from 0.72 to 0.79. For the most important temperature range 30–40°C, one can assume the value of Pr = 0.75, which is average for it. On the contrary, the Pr_d number in the 0–70°C range remains practically constant and equal to 0.62. The diffusion coefficient of water vapors D depending on the temperature is determined by the formula of Fuller et al. [10], which is the most reliable formula of the known ones:

$$D = \frac{2\cdot 10^{-7}\cdot T^{1.75}\left(\dfrac{1}{M_w} + \dfrac{1}{M_{\mathrm{air}}}\right)^{0.5}}{p\,(12.7^{1/3} + 20.1^{1/3})^2}\,,$$

where p is the pressure, atm; M_w and M_{air} are, respectively, the molecular weight of the water and the air. Thus, the mean Lewis number $Le = Pr_d/Pr = 0.62/0.75 = 0.83$, which points to the absence of strict analogy between the processes of heat and mass transfer in the gas phase usually taken in calculations.

Now all functions and parameters entering into Eqs. (6), (11), and (12) have been determined. For their unique solution, one has to know the changes in the temperature at the interface and in the equilibrium concentration C_i that are related by the third boundary condition in (4).

In the present paper, we use the approach based on the approximate solutions of Eqs. (6), (11), and (12) at constant values of t_i and C_i with subsequent use of the superposition method (Duhamel theorem in heat conduction), since these equations are linear. We find their approximate solution under constant boundary conditions of the first kind by the two-parameter integral method described in detail in [6]. In this monograph, a high efficiency and an accuracy of the method comparable to the accuracy of the best numerical and available exact methods was shown with many examples of solving linear and nonlinear boundary-value problems for boundary conditions of the I–IV kinds under laminar and turbulent liquid flows. However, as opposed to the latter, the advantage of this method is the fact that it permits obtaining analytical solutions in closed form.

Let us first turn to the dimensionless temperatures and concentration. For Eq. (6) we introduce $t^* = \dfrac{t - t_{in}}{t_i - t_{in}}$, where t_{in} is the temperature of the inflowing hot water, and for (11) and (12) — $T^* = \dfrac{T - T_{in}}{T_i - T_{in}}$ and $C^* = \dfrac{C - C_{in}}{C_i - C_{in}}$, where T_{in} and C_{in} are, respectively, the temperature of the inflowing cold air and the initial concentration of vapor in it. If we omit the sign * in the dimensionless temperatures and concentration, then Eqs. (6), (11), and (12) will preserve their form, and we shall write the boundary conditions as

$$t(0, \eta) = 0\,, \quad t(\xi, 0) = 1\,, \quad \partial t(\xi, 1)/\partial \eta = 0\,. \tag{14}$$

Analogous conditions are also written for Eqs. (11) and (12) upon replacement of t by T or C.

Consider sequentially the solutions of Eqs. (6), (11), and (12) under the boundary conditions (14). The solution of problem (6), (14) is given in [6] with the example of mass transfer (pp. 199–205). Therefore, we give here only the finite dependences for the thermal initial and stabilized regions of the heat exchange. The dimensionless temperature profiles in these regions of the heat exchange are found in the form

$$t = (1 - \eta/q)^{n_1}\,, \quad t = (1 - t_2)\,(1 - \eta)^{n_2} + t_2\,, \tag{15}$$

where q is the dimensionless thickness of the thermal boundary layer; t_2 is the dimensionless temperature on the solid wall; n_1 and n_2 are the profile parameters (degrees). The dependences of q, n_1, n_2, and t_2 are determined in the process of solving the problem of two equations. One of them is the first integral of Eq. (6) and the second one is obtained by multiplying (6) by t and integrating it within the thermal boundary layer. All substantiations of the method are given in [6], and the detailed procedure of finding q, n_1, n_2, and t_2 is described in [11] with the example of the heat exchange in a flowing liquid film in the thermal initial region under boundary conditions of the second kind.

As a result, it has been found that for the first and second regions of heat exchange $n_1 = n_2 = 1.8$ can be assumed. Then for q and t_2 we have the following explicit relations:

$$q = 3.175\xi^{1/2} + 0.261\xi^{3/2}\,,$$

$$t_2 = 1 - \exp\left[-\,5.162\,(\xi - \xi_1)\right]\,, \tag{16}$$

where ξ_1 is the dimensionless length of the thermal initial region, i.e., the ξ value at $q = 1$. From (16) we find that $\xi_1 = 0.0976$. Let us estimate the length of the thermal region in the physical variables for $t_{av} = 30^{\circ}$C, $Pr = 5.42$ at the maximum density of the laminar-wave flow, i.e., at $Re_f = 400$. Then $Pe = 3252$, $\delta = 1.44(\nu/g)^{1/3}$, $\nu = 0.81 \cdot 10^{-6}$ m^2/sec, $\delta = 0.4307 \cdot 10^{-3}$ m, and $x_1 = \xi_1 \delta Pe = 0.137$ m. The local and mean Nusselt numbers are determined by the relations

$$\mathrm{Nu} = -\frac{\partial t(\xi, 0)}{\partial \eta} (1 - t_{\mathrm{av}})^{-1}, \quad \overline{\mathrm{Nu}} = \frac{1}{\xi} \int_0^{\xi} \mathrm{Nu}\, d\xi . \tag{17}$$

Adequate substitutions yield the following dependences:

$$\mathrm{Nu}_1 = \frac{1.8}{q} [1 - 0.536q + 0.0587q^3]^{-1} ,$$

$$\overline{\mathrm{Nu}}_1 = -\frac{0.667}{\xi} \ln [1 - 0.536q + 0.0587q^3] = 4.434 , \tag{18}$$

$$\mathrm{Nu}_{\mathrm{wall}} = 3.442 , \quad \overline{\mathrm{Nu}}_2 = \frac{4.434 \cdot 0.0976 + 3.442\,(\xi - 0.0976)}{\xi} , \quad \xi > \xi_1 .$$

Estimating the value of the film-to-air heat-transfer coefficient for limiting $\mathrm{Nu}_{\mathrm{wall}} = 3.442 = \alpha\delta/\lambda$, under the above conditions we have $\alpha = 4.939 \cdot 10^3$ W/(m$^2\cdot{}^{\mathrm{o}}$C). Likewise, we obtain the solution of problem (11), (14) in the thermal initial and stabilized regions of the heat exchange, where the temperature profiles T are also sought in the form of (15).

Details of the solution method for the case of the heat exchange of the turbulent flowing liquid film under boundary conditions of the first and second kinds can be found in [6, 12, 13]. As a result, for the thermal initial region it has been found that $n_1 = 1.7578$ at $q \le 7.8/(h^+ - \delta^+)$ (laminar viscous sublayer), and for the region of $7.8/(h^+ - \delta^+) < q \le 1$ we have obtained the equation

$$\frac{(n_1 + 2)\,[(n_1 (1 - s) + 1 - s/2]}{2\,(2n_1 + 1)\,[n_1 (1 - s) + 2 - s]} = \psi (n_1, q) ,$$

where

$$\psi (n_1, q) = \frac{n_1 - 1}{2n_1 - 1} - bn_1 q \left[(h^+ - \delta^+ + 7.8) \left(\frac{\varphi^{2n_1 - 1}}{2n_1 - 1} - \frac{\varphi^{2n_1}}{2n_1} \right) - \right.$$

$$\left. - (h^+ - \delta^+)\, q \left(\frac{\varphi^{2n_1 - 1}}{2n_1 - 1} - \frac{\varphi^{2n_1}}{n_1} + \frac{\varphi^{2n_1 + 1}}{2n_1 + 1} \right) - \frac{7.8\varphi^{2n_1 - 1}}{2n_1 - 1} \right] ; \tag{19}$$

$$b = 0.433\mathrm{Pr} ; \quad \varphi = 1 - \frac{7.8}{q\,(h^+ - \delta^+)} \frac{1}{q} .$$

With increasing q the function φ approaches unity, which considerably simplifies the calculations in obtaining the parameter n_1 from (19).

The dimensionless thickness of the thermal boundary layer is found by the formula

$$q = (A_2 \xi_{\mathrm{g}})^{\frac{1}{2+s}} , \quad A_2 = \frac{n_1 (n_1 + 1)(n_1 + 2)(2 + s)}{(1 + s)\,[n_1 (1 - s) + (2 - s)]} . \tag{20}$$

Determining the average dimensionless temperature of air in the section as

$$\overline{T} = \frac{1}{\overline{u}_{\mathrm{g}0}} \int_0^1 u_{\mathrm{g}0} T d\eta_{\mathrm{g}} , \quad \overline{u}_{\mathrm{g}0} = \int_0^1 \eta_{\mathrm{g}}^s d\eta_{\mathrm{g}} = \frac{1}{1 + s} ,$$

we find for this region

$$\overline{T}_1 = \frac{(1+s)\,[n_1\,(1-s)+(2-s)]}{(n_1+1)\,(n_1+2)}\,q^{1+s}\,. \tag{21}$$

For the stabilized region the following coupling equation for the parameter n_2 has been obtained:

$$\frac{(1+s)\,(2n_2+1)\,[n_2\,(1-s)+2-s]-(1+s)\,(n_2+2)\,[n_2\,(1-s)+1-s/2]}{(2n_2+1)\,[(n_2+1)\,(n_2+2)-(1+s)\,(2-s)]} =$$

$$= \frac{n_2-1}{2n_2-1} - \frac{b\,(h^+-\delta^+)}{2\,(2n_2+1)}\left(1-\frac{7.8}{h^+-\delta^+}\right)^{2n_2+1}\,. \tag{22}$$

As is seen from (19) and (22), the parameters n_1 and n_2 depend on Re_{g0} through $h^+-\delta^+$ (most strongly), the index s in the velocity profile, and the Pr number. Moreover, n_1 depends on q. The exponent ($s<1/7$), in general, produces a small effect on the values of n_1 and n_2, leading to their decrease by 1% as a maximum.

The dimensionless temperature on the channel axis T_2 is determined by the dependence

$$T_2 = 1-\exp\,[-A_3\,(\xi_g-\xi_{g1})]\,,\quad A_3 = \frac{n_2}{\dfrac{1}{1+s}-A_4}\,,$$

$$A_4 = \frac{n_2\,(1-s)+(2-s)}{(n_2+1)\,(n_2+2)}\,. \tag{23}$$

The average dimensionless temperature in this region is calculated by the relation

$$\overline{T}_2 = (1+s)\,A_4\,(1-T_2)+T_2\,. \tag{24}$$

The local Nu_g numbers in the first and second regions are determined by the dependences

$$\mathrm{Nu}_{g1} = \frac{n_1}{q}\left(1-A_5 q^{1+s}\right)^{-1}, \tag{25}$$

$$A_5 = \frac{(1+s)\,[n_1\,(1-s)+(2-s)}{(n_1+1)\,(n_1+2)}\,,$$

$$\mathrm{Nu}_{g,\mathrm{wall}} = \frac{n_2}{1-A_5'}\,,\quad \mathrm{Nu}_g = \frac{\alpha_g\,(h-\delta)}{\lambda_g}\,, \tag{26}$$

where A_5' is determined by analogy with A_5 by replacing n_1 with n_2.

The mean $\overline{\mathrm{Nu}}_g$ numbers in the first and second regions are calculated as follows:

$$\overline{\mathrm{Nu}}_{g1} = -\frac{1}{(1+s)\,\xi_g}\ln\,(1-\overline{T}_1)\,, \tag{27}$$

$$\overline{\mathrm{Nu}}_{g2} = [\overline{\mathrm{Nu}}_{g1}\,\xi_{g1}+\mathrm{Nu}_{g,\mathrm{wall}}\,(\xi-\xi_{g1})]/\xi_g\,, \tag{28}$$

where $\overline{\mathrm{Nu}}_{g1}$ is the value of the mean $\overline{\mathrm{Nu}}_g$ at the end of the first region.

Finally, the solution of the mass-transfer problem (12) with boundary conditions analogous to (14) is described by the same relations as for the heat-exchange problem (11), (14), i.e., (19)–(28). The only difference is that now the diffusion number $\mathrm{Pr_d}$ will enter into (19) and (22) instead of the thermal number Pr, and in (20) and (23) thereby one should use the coefficients A_2/Le and A_3/Le.

We emphasize that under a turbulent flow of liquids the known fact is practical independence of the Nusselt numbers of the change in the heat-exchange surface temperature along the channel lengths [14]. In [6], this was confirmed for the heat exchange in a turbulent film. By analogy, the independence of the Nu numbers on the temperature change at the interface is often used to simplify the corresponding calculations of the two-phase heat and mass transfer. However, the heat and mass-transfer coefficients are not so universal, which is their great disadvantage, although they are used in all existing computing methods for cooling towers.

Using the solutions obtained, let us make some important estimates, assuming the channel half-width h = 0.025 m and $\delta = 0.4307\cdot10^{-3}$ m (at $\mathrm{Re_f} = 400$ the gas velocity $\bar{u}_\mathrm{g} = 4$ m/sec). In so doing, the velocity on the film surface according to the experiment of [9] in the second wave laminar regime is determined by the formula

$$u_i = 0.485\,(\nu_\mathrm{liq} g)^{1/3} 4^{0.6} \mathrm{Re_f^{0.6}}\,. \tag{29}$$

For the average temperature of the liquid $t_\mathrm{av} = 30^\mathrm{o}\mathrm{C}$, we obtain $u_i = 0.81$ m/sec, the mean relative velocity of the gas flow $\bar{u}_\mathrm{g0} = \bar{u}_\mathrm{g} + u_i = 4.81$ m/sec, the Reynolds number for the relative velocity $\mathrm{Re_{g0}} = 4\bar{u}_\mathrm{g0}(h-\delta)/\nu_\mathrm{g} = 3.06\cdot10^4$, $\lambda_0' = 2.392\cdot10^{-2}$, the drag coefficient at the interface is equal to $\lambda_0 - \lambda_0' = 5.694\cdot10^{-3}$, and then $\lambda_0 = 2.961\cdot10^{-2}$. The dynamic velocity is determined as $V_\mathrm{g0}^* = \bar{u}_\mathrm{g0}\sqrt{\lambda_0/18} = 0.293$ m/sec. The dimensionless half-width of the channel and the film thickness are equal to $h^+ = hV_\mathrm{g0}^*/\nu_\mathrm{g} = 473.8$, and $\delta^+ = \delta\vartheta_\mathrm{g0}^*/\nu_\mathrm{g} = 8.16$, which constitutes 1.72% of h^+. The maximum velocity value, according to (10), is $u_\mathrm{g0max} = 6.002$ m/sec. The exponent value in the velocity profile, according to (8), is $s = 0.1463$, i.e., $1/s = 6.837 < 7$.

We consider next the stabilized regions of the heat and mass transfer in the gas phase. Let us find the main characteristics — the parameters of profiles n_2 and n_2d. The input equation is (22). Using the method of successive approximations, we obtain the following results: for the heat transfer at $\mathrm{Pr_g} = 0.75$ $n_2 = 28.72$, and for the mass transfer at $\mathrm{Pr_d} = 0.62$ $n_\mathrm{2d} = 26.35$, which is 9% lower than the value for the heat transfer. Using them, we determine the values of $\mathrm{Nu_{wall}}$ from (26): $\mathrm{Nu_{g,wall}} = 29.704$, $\mathrm{Nu_{d,wall}} = 27.334$.

Let us write the ratio between the thermal and diffusion Nusselt numbers in the expanded form:

$$\frac{\mathrm{Nu_{g,wall}}}{\mathrm{Nu_{d,wall}}} = \frac{\alpha_\mathrm{g} D}{\beta_\mathrm{g}\lambda_\mathrm{g}} = \frac{\alpha_\mathrm{g}}{\beta_\mathrm{g}}\,\frac{D}{a_\mathrm{g}\rho_\mathrm{g}c_{p\mathrm{g}}} \quad \text{or} \quad \frac{\alpha_\mathrm{g}}{\beta_\mathrm{g}} = \frac{\mathrm{Nu_{g,wall}}}{\mathrm{Nu_{d,wall}}}\,\frac{\mathrm{Pr_d}}{\mathrm{Pr_g}}\,\rho_\mathrm{g}c_{p\mathrm{g}}\,. \tag{30}$$

Substituting into (30) concrete values of Nu and Pr, we get

$$\frac{\alpha_\mathrm{g}}{\rho_\mathrm{g}c_{p\mathrm{g}}} = 0.9\beta_\mathrm{g}\,. \tag{31}$$

Thus, the difference from the known Lewis relation relating the heat-transfer coefficients α_g and the mass-transfer coefficients β_g is 10% on average. Because of the different values of Pr, this difference can also reach 15–20%.

Estimating the absolute value of stabilized α_g, we find $\alpha_\mathrm{g} = \mathrm{Nu_{g,wall}}\lambda_\mathrm{g}/(h-\delta) = 31.68$ W/(m$^{2}\cdot{}^\mathrm{o}$C). Determine the length of the thermal initial region from (20), assuming $q = 1$ and $n_1 = n_2$. From the relation $\xi_\mathrm{g} = x_\mathrm{g}/(h-\delta)$ $\mathrm{Pe_g}$, taking into account that $\mathrm{Pe_g} = \mathrm{Re_{g0max}}\,\mathrm{Pr_g} = 7257$, we find $\xi_\mathrm{g} = x_\mathrm{g}/178.3$. From formula (20), we have $\xi_\mathrm{g,wall} = 1/A_2$, $A_2 = 1861.7$. Then $\xi_\mathrm{g,wall} = 5.372\cdot10^{-4}$, $x_\mathrm{g,wall} = 178.3\xi_\mathrm{g,wall} = 0.0958$ m.

Thus, the length of the thermal region for a turbulent air flow is small compared to the real length of the heat-transfer surface, measuring 3–4 m.

Of great interest is comparison between theoretical and experimental data for the heat and mass-transfer coefficients assigned to the surface. Note that the running of such experiments is rather complicated and laborious and, therefore, there have been very few. Berman [2] staged experiments under the above conditions of a turbulent air

TABLE 1. Values of $\mathrm{Nu}_{\mathrm{d,wall}}$ and β_{g} for the Case of $\bar{t} = 30^{\circ}$C and $\bar{T} = 20^{\circ}$C at Various Air Velocities

$\bar{u}_{\mathrm{g}}$, m/sec	$\bar{u}_{\mathrm{g0}}$, m/sec	n_{2d}	$\mathrm{Nu}_{\mathrm{d,wall}}$	$\beta_{\mathrm{g}} \cdot 10^2$, m/sec	$\beta_{\mathrm{g,exp}} \cdot 10^2$, m/sec
1	1.81	11.38	12.366	1.23	1.193
2	2.81	16.6	17.584	1.748	1.752
3	3.81	21.525	22.509	2.23	2.37
4	4.81	26.35	27.334	2.72	2.74

TABLE 2. Values of t_2^* and t_{av}^* at Some Values of the Dimensionless Longitudinal Coordinate ξ ($\xi = x/1.4017$)

x, m	ξ	t_2^*	t_{av}^*
0.5	0.3567	0.7342	0.861
1.0	0.7133	0.958	0.978
2.0	1.4266	0.999	0.9995

counterflow and a laminar flow of a water film by solid boards. The results of the experiments are presented in [2] in the form of a diagram of the values of the mass-transfer coefficients (evaporation coefficients) β_p assigned to the partial pressure difference depending on the relative air velocity. We have performed theoretical calculations throughout the region of turbulent air flow corresponding to the experiments ($\bar{u}_{\mathrm{g}} \sim$ 1–4 m/sec). In so doing, the β_p values were taken from the graphs and transferred to the β_g values assigned to the volume concentrations. A practically complete quantitative and qualitative agreement between the theoretical and experimental data has been found. As an example, we give the values of $\mathrm{Nu}_{\mathrm{d,wall}}$ and β_{g} for the case of $\bar{t} = 30^{\circ}$C and $\bar{T} = 20^{\circ}$C considered in the experiment for various air velocities (Table 1). Characteristically, the dependence of $\beta_{\mathrm{g}} \sim \bar{u}_{\mathrm{g0}}^{0.842}$ practically coincides with the experimental dependence $\beta_{\mathrm{g,exp}} = \bar{u}_{\mathrm{g0}}^{0.8}$.

Thus, the account of the main features of the process has for the first time made it possible to find the theoretical relations for the heat and mass-transfer coefficients corresponding to the experiment but being much more general and informative. As would be expected, because of the small Pr numbers the influence of waves on the process of heat and mass transfer is insignificant. This is in good agreement with the classification of the processes by the transfer models formulated in [6].

Consider the case of heat and mass transfer where the heat flows at the interface turn out to be related by the last equation in [4], defining the temperature t_i and its change along the length of the heat-transfer surface. We shall take into account the values of the coefficients of heat transfer from the liquid film to the gas, as well as the fact that in industrial cooling towers the liquid is cooled in a narrow temperature range (10–12°C as a maximum), and, therefore, we can take t_i to be equal to the liquid film surface temperature. In this connection, let us estimate the change in the temperature across the film.

The average dimensionless temperature of the film with allowance for t_2^* according to (16) is determined as

$$t_{\mathrm{av}}^* = 1 - 0.523 \exp\left[-5.162\left(\xi - 0.0976\right)\right] .$$

Let us find t_2^* and t_{av}^* at some values of the dimensionless longitudinal coordinate $\xi = x/1.4017$ (Table 2).

Thus, a certain nonuniformity of the temperature profile is observed approximately up to the value of $x = 0.5$ m. However, since the temperature pressures $t_i - t_{\mathrm{in}}$ and $t - t_{\mathrm{in}}$ turn out to be low, the degree of supercooling of the film surface with respect to t_{av} is also small. The foregoing permits considering the temperature t_i for the gas to be equal to t_{av} for the liquid and linearly changing along the length, i.e., determining it by the expression

$$t_i = t_{\mathrm{fin}} + \frac{t_{\mathrm{in}} - t_{\mathrm{fin}}}{\xi_{\mathrm{g}l}} \xi_{\mathrm{g}} = t_{\mathrm{fin}} + a_2 \xi_{\mathrm{g}} , \quad a_2 = \frac{t_{\mathrm{in}} - t_{\mathrm{fin}}}{\xi_{\mathrm{g}l}} , \tag{32}$$

where t_{fin} is the finite temperature of the liquid; ξ_{gl} corresponds to $x = l$.

The equilibrium value of the concentration C_i in the 15–20°C temperature range can also be linearly related, to a high accuracy, to the temperature

$$C_i = a_3 + b_3 t_i \,, \tag{33}$$

where the coefficients a_3 and b_3 are determined from the thermodynamic tables. Substituting (32) into (33), we also obtain a linear dependence of C_i on the longitudinal coordinate:

$$C_i = a_4 + b_4 \xi_2 \,, \quad a_4 = a_3 + b_3 t_{\text{fin}} \,, \quad b_4 = a_2 b_3 \,. \tag{34}$$

Using (32) and (34), we apply the superposition method to the solutions obtained above, and it is most convenient to apply it at once to the heat and mass gas flow densities entering into the right-hand side of (4).

As a result, we have [14]

$$q_{\text{g}} = - \lambda_{\text{g}} \left. \frac{\partial T}{\partial y_{\text{g}}} \right|_{y_{\text{g}}=0} , \quad m_{\text{v}} = - D \left. \frac{\partial C}{\partial y_{\text{g}}} \right|_{y_{\text{g}}=0} ,$$

$$q_{\text{g}} = - \frac{\lambda_{\text{g}}}{h - \delta} \left. \frac{\partial T}{\partial \eta_{\text{g}}} \right|_{\eta_{\text{g}}=0} , \quad m_{\text{v}} = - \frac{D}{h - \delta} \left. \frac{\partial C}{\partial \eta_{\text{g}}} \right|_{\eta_{\text{g}}=0} . \tag{35}$$

Substituting the known expressions for the derivatives into (35), we obtain

$$q_{\text{g}} = \frac{\lambda_{\text{g}}}{h - \delta} \left[\int\limits_0^{\xi_{\text{g}}} n_2 \, (1 - T_2) \, (\xi_{\text{g}} - \tau) \, \frac{dt_i}{d\tau} \, d\tau + \Delta t_i n_2 \, (1 - T_2) \right] , \tag{36}$$

$$m_{\text{v}} = \frac{D}{h - \delta} \left[\int\limits_0^{\xi_{\text{g}}} n_{2g} \, (1 - T_{2g}) \, (\xi_{\text{g}} - \tau) \, \frac{dC_i}{d\tau} \, d\tau + \Delta C_i n_2 \, (1 - T_{2g}) \right] . \tag{37}$$

where τ is the integration variable: $\Delta t_i = t_i - T_{\text{in}}$; $\Delta C_i = C_i - C_{\text{in}}$; ξ_{g} is considered to be constant.

Calculate the integrals in (36) and (37) in view of (23). The equations obtained are integrated again to obtain the total heat and mass flows along the full length, which makes it possible to find the gas temperature and humidity at the outlet, as well as to estimate the water cooling. If it disagrees with the given value or the air humidity is higher than the humidity upon saturation, then it is necessary to perform similar calculations with a higher gas velocity.

Thus, the proposed approach enables one to make calculations be transparent and convenient for engineering analysis, represent them in analytical form, and consider a wide variety of cases under different initial conditions of the air.

NOTATION

a, thermal diffusivity, m^2/sec; c_p, heat capacity, J/(kg·$^{\text{o}}$C); C, local concentration of water vapor in the air, kg/m^3; D, diffusion coefficient of water vapor in the air, m^2/sec; g, acceleration of gravity, m/sec^2; h, channel half-width, m; h^+, dimensionless half-width of the channel; l, board length, m; Le = a/D, Lewis number; m_{v}, mass flow of water vapor, kg/(m^2·sec); n_1 and n_2, profile parameters; Nu, Nusselt number; Δp, pressure drop; Pe = Re Pr, Peclet number; Pr = ν/a, Prandtl number; q, dimensionless thickness of the thermal boundary layer; r, phase-transition heat, J/kg; Re, Reynolds number; t, local film temperature, $^{\text{o}}$C; T, local air temperature, $^{\text{o}}$C; u and u_{av}, local and average velocity of the film, m/sec; $u_{\text{g}0}$, local air velocity, m/sec; $\overline{u}_{\text{g}0}$, average relative air velocity, m/sec; V, dynamic velocity, m/sec; x and y, longitudinal and transverse coordinates, m; α, heat-transfer coefficient, W/(m·$^{\text{o}}$C); β, mass-transfer coefficient, m/sec; δ, mean film thickness; δ^+, dimensionless film thickness; λ, heat-conductivity coefficient, W/(m·$^{\text{o}}$C); λ_0 and λ_0', drag coefficients of the sprayed and smooth pipe; ν, kinematic viscosity, m^2/sec; ρ, density, kg/m^3. Subscripts: d, diffusion; *i*, interface; g, gas; f, film; t, turbulent; in, initial; fin, finite; max, maximum value; bar, mean value; av, average; v, vapor; wall, wall; exp, experimental; w, water; air, air; liq, liquid.

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